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An Integrated Fuzzy DEMATEL-ANP-TOPSIS Methodology for Supplier Selection Problem

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ABSTRACT

In recent changing and dynamic environment because existing competition and need for cooperation among companies, supplier selection has become as a critical strategic factor for most company. The supplier selection is a complex multi-criteria problem containing both quantitative and qualitative criteria with complex relationships which may have trade-offs and interaction than each other. In the paper, a novel hybrid multi-criteria decision making (MCDM) methodology is proposed to encompass the complex relationships and cope with the trade-offs among criteria. Given those ambiguous or uncertain criteria, it is essential that fuzzy approach is employed for handling the uncertainty and achieving more accurate results. The ANP method used for determining the appropriate weightings to each sub-criterion was developed to overcome the problems of dependence and feedback among criteria. For identifying relationship network among criteria, DEMATEL method is employed as a supportive tool for ANP. Then TOPSIS is used to rank all competing alternatives according their performance. In MCDM studies, applying TOPSIS in ranking alternatives has recently been customary because of its advantages in solving problems with trade-offs-including criteria. In the end, a numerical example is demonstrated to show the proposed model can improve solving of supplier selection problem.

1. Introduction

In manufacturing companies, the role of purchasing and outsourcing of raw materials and component parts is very important and play a vital role on their success and competitiveness and without it companies cannot survive and continue their activities. One of the most critical activities in purchasing management is supplier selection including a strategic decision for companies' survival. Supplier selection and evaluation continues to be a key element in the industrial buying process and appears to be one of the major activities of the professional industrial [1]. Given that raw material and component parts purchased from outside suppliers consist of considerable portion of the expenses, those must be bought from suppliers providing products with reasonable prices and favorable quality in the right place and the time. For instance in most automotive companies, these costs exceed 50 percent of sales [2]. In other hand, Weaknesses in the procurement of the materials and parts can cause unemployment of other assets and increasing costs and ultimately reduce profitability and even losses. Thus, almost in all companies, the purchase from outside is considered as a critical responsibility. Selection of the incorrect supplier could upset the company's financial and operational condition, while the selection of an appropriate supplier may significantly reduce the purchasing cost and improve competitiveness. In the outsourcing operation model, supplier selection is one of the critical factors affecting the final success. Also the supplier selection problem has been widely studied and determined as a multiple criteria decision making (MCDM) issue [3]. Therefore, supplier selection problem is a kind of multiple criteria decision making problem which requires MCDM methods for solutions with high accuracy. Due to nature of the problem including multiple and usually unstructured criteria, the techniques of MCDM can be coherently applied as a decision making tool. In earlier studies, Multi-criteria decision making (MCDM) models have been widely utilized for the supplier selection problem. Ghodsypour and O'Brien [4] used an integrated AHP and LP approach for the supplier selection and argued that AHP is more precise than other ranking methods. Tam and Tummala [5] applied an AHP-based model to a real case and indicate that it can improve the group decision making in supplier selection problem. In spite of the AHP considerable application, it has been recognized that always is not suitable for solving the various problem that involves intertwined evaluation criteria because each individual criterion may completely be depended to other ones complicating evaluation. In other word, when AHP should be used in the decision-making that exist a unilateral hierarchical relationship among decision levels [6]. Carney and Wallnau [7] argued that the evaluation criteria for alternatives in complex environments are not always independent of each other, but can include interdependence and feedback among themselves. For overcoming the interdependence among criteria, a considerable number of studies have developed decision making models based on the MCDM approach. Among the methods, the ANP has been widely considered as an appropriate decision making method. Lin et al. [3] employed the ANP method for solving supplier selection problem for a semiconductor company. For coping with the complex and interactive relation among attributes, they also used interpretive structural modeling (ISM) to determine the structural relationships and the interrelationships amongst all the evaluation's dimensions. In real world, purchasing decision-making includes factors and criteria that their available information in a MCDM process is usually vague and imprecise. Zadeh in 1965 first proposed fuzzy set theory which provided a framework for solving problems in fuzzy environments. Fuzzy set theory is useful when the purchase situation is full of uncertainty and imprecision due to the human judgments making the decision very complex and unstructured. Some researchers applied fuzzy set theory to solve the supplier selection problem considering uncertainty. De Boer et al. [8] by providing a comprehensive review of the supplier selection's literature proposed the fuzzy set theory as a way for improving the supplier selection process. In addition, to find the supplier with the best overall performance rating among suppliers, Erol et al. [9] discussed the advantages of fuzzy set theory in supplier selection issues. Also recently Kumar et al. [10] have applied a fuzzy goal programming approach for solving the supplier selection problem in supply chain providing a decision method for handling the vagueness and imprecision objectives. Shemshadi et al. [11] for solving supplier selection problem extended the fuzzy VIKOR method with a mechanism to extract and deploy objective weights based on Shannon entropy concept. Sanayei et al. [12] proposed a hierarchy MCDM model based on fuzzy sets theory and VIKOR method to deal with the supplier selection problems in the supply chain system. In order to better solve the above-mentioned problems, this paper proposes a hybrid novel MCDM model that can provide a solution with better quality. This paper is different from previously research in three ways. First, we adopt DEMATEL to identify the relationship network among the criteria. Second, the weights of each criterion can be determined using the fuzzy ANP method. Third for ranking all competing alternatives, The TOPSIS is used as suitable method. Finally, given that uncertain data, we obtain the overall scores of each supplier in a fuzzy environment. To obtain the sub-criteria weightings, FANP (Fuzzy Analytic Network Process) method is employed which are able to overcome the problem of interdependence and feedback amongst criteria. The supplier selection problem is an unstructured, complicated, and multi- criteria decision problem [13]; Here in ANP to determine the structural relationships and the interrelationships among all criteria, the decision making trial and evaluation laboratory (DEMATEL) method is employed as depict interrelations map. In recent years, the DEMATEL has become very popular because it can visualize the structure of complicated causal relationships. Then alternatives ranking should be determined which can assist the decision making. For the purpose, various techniques can be applied as ranking tool such as TOPSIS, ELECTRE and VIKOR. For the purpose, The TOPSIS is used to rank all competing alternatives in terms of their overall performances. Here, given that the effect of each attribute is not always unilateral and must be considered as a trade-off in term of other attributes, TOPSIS can help the decision makers to reach a decision with high quality; Such problems are often complicated because the identification and analysis of tangible and intangible multiple criteria. The TOPSIS is suitable and widely applied as technique for solving MCDM problems based on the concept that the optimal alternative should have the shortest distance from the positive idea solution and the farthest distance from the negative idea solution (14). In other hand, because data related the criteria of supplier selection issues are ambiguous and uncertain, the fuzzy approach is employed for coping with the uncertainty and attaining more accurate results. Therefore, in present study, According to the characteristics of the problem and the techniques, we will establish a hybrid model for supplier evaluation for solving the optimization problem for supplier selection problem including DEMATEL, ANP and TOPSIS. By combining the above three techniques, we can provide a suitable way for properly selecting supplier in a fuzzy environment. This paper also conducted a numerical example as an illustration to demonstrate how a company can implement this model.

2. Fuzzy set theory

Fuzzy set theory first was introduced by Zadeh (1965) to map linguistic variables to numerical variables within decision making processes. Fuzzy set theory first was introduced by Lotfi Zadeh [15] to explain uncertainty in events and systems where uncertainty arises due to vagueness. Bellman and Zadeh (1970) [16] introduced fuzzy multicriteria decision making (FMCDM) methodology to resolve the lack of precision in determining importance weights of criteria and the ratings of alternatives regarding evaluation criteria. Given that data on the multicriteria decision making problem stated by different experts is ambiguous and vague, applying linguistic terms is necessary to cope with the situations. A linguistic variable is one whose values are linguistic terms, i.e. sentences is to easily express the imprecision qualitative of an experts assessments [17]. For example, service satisfaction is a linguistic variable as its linguistic terms can be ‘‘very poor”, ‘‘poor”, ‘‘fair”, ‘‘good”, and ‘‘very good”. Each linguistic variable can be represented by a fuzzy number which can be assigned to a membership function. Among fuzzy numbers, triangular fuzzy numbers have been identified as useful means of quantifying the uncertainty in decision making because of their intuitive appeal and efficiency in computation [18]. A positive triangular fuzzy number $\tilde{A}$ can be denoted as $\tilde{A} = (a_1, a_2, a_3)$ where $(a_1 \geq a_2 \geq a_3)$, $a > 0$, and if $a_1 = a_2 = a_3$, “*A*” cannot be called a fuzzy number anymore. A fuzzy number $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$. The membership function $\mu_{\tilde{A}}(x)$ quantifies the grade of membership of the element *x* to the fuzzy set $\tilde{A}$ defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} (x - a_1) / (a_2 - a_1), & a_1 \leq x \leq a_2, \\ (x - a_2) / (a_3 - a_2) & a_2 \leq x \leq a_3, \\ 0 & otherwise, \end{cases} \qquad (1)$$

Also the crisp value of the fuzzy number $\tilde{A}$ based on center of gravity (COG) method can be expressed by following relation:

$$\bar{c}(\tilde{A}) = \frac{\int_a^d x\, \mu_{\tilde{A}}(x)dx}{\int_a^d \mu_{\tilde{A}}(x)dx} = \frac{1}{3}\left[a_1 + a_2 + a_3\right], \qquad (2)$$

In our approach, the linguistic scale referring to the importance of the criteria and the ratings of the alternatives are made depending on the scale of 5 points and 7 points, respectively, shown in Tables 1 and 2.

Table 1 linguistic variable for pairwise comparison of criteria

Linguistic scale for importance	Triangular fuzzy number
Equally important	(1, 1, 1)
Weak importance	(2, 3, 4)
Strong importance	(4, 5, 6)
dominant importance	(6, 7, 8)
Absolute importance	(8, 9, 10)

Table 2 Linguistic variables for performance rating of alternatives

Linguistic scale for importance	Triangular fuzzy number
Very poor	(0, 0, 0.1)
Poor	(0, 0.1, 0.3)
Medium poor	(0.1, 0.3, 0.5)
Fair	(0.3, 0.5, 0.7)
Medium good	(0.5, 0.7, 0.9)
Good	(0.7, 0.9, 1)
Very good	(0.9, 1, 1)

3. Methods

DEMATEL:

DEMATEL method is introduced to build the structure of relationship map for clarifying the interrelations among criteria, as well as to visualize the causal relationship of criteria through a causal diagram. The diagraph depict a basic concept of contextual relation among the criteria of the system, in which the numeral represents the strength of influence. This is a decision making method in the case that several criteria have complex relationships. The purpose of this technique is to study complex issues, which analyze them and create a network structure based on this analysis. Because of existence of impact among capabilities, this method is employed to extract the interdependencies among them and the strength as well. This eventually determines casual and effect relationships between these criteria and shows indirect effects of ones on each other's, which can improve results of the ANP technique [19].

ANP:

The ANP, developed by Thomas L. Saaty, is an extension of analytic hierarchy process (AHP). The analytic hierarchy method (ANP) allows for complex interrelationships among decision levels and attributes [20]. ANP is a comprehensive decision making technique that measure the dependence and feedback within and between the criteria or alternatives. The ANP has been used for the decision making under multiple criteria to remove the restriction of hierarchical structure, and has been employed for the selection problems. In our proposed model, FANP will be used only to calculate the weights for the relative importance of the sub-criteria applied to support fuzzy TOPSIS for ranking the alternatives.

TOPSIS:

We apply the TOPSIS method to calculate the overall score for each alternative. According to this method, the alternative with minimum distance from the positive- ideal solution and greatest distance from the negative ideal solution would be best one [21]. The characteristics of the TOPSIS method make it an appropriate approach which has good potential for solving selection and evaluation problems including [22]:

(1) Employing TOPSIS reduce the number of pair-wise comparisons and can include infinite range of alternative properties and performance attributes;

(2) In the context of vender selection issues, because the effect of each attribute is not always unilateral and must be considered as a trade-off in term of other attributes, the TOPSIS can be an appropriate method. Many manufacturing managers believe there is trade-off between cost, delivery, flexibility, and service features in the supplier selection issues for raw materials and component parts;

(3) The output can be determined numerically, a preferential ranking of the alternatives (candidate supplier), that better show differences and similarities between alternatives, whereas other MADM techniques such as the ELECTRE method only can determine the priority of each supplier.

4. The proposed method for supplier selection

In this section we are going to propose supplier selection algorithm that supports subjective and objective weights. The problem of supplier selection in supply chain system could be treated as a group multiple criteria decision making (GMCDM) problem, which could be described as followings:

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Pre-phase: identifying conflicting criteria and competing alternatives

Step 1. Company desires to select a suitable supplier to purchase the key components of its new product and form a decision committee for the aim.
Step 2. Arrange the decision making group and define and describe a finite set of relevant attributes. Moreover, candidate alternatives are selected for further evaluation based preliminary screening.
Step 3. Identify and define linguistic terms and relevant membership functions. Two set of appropriate linguistic variables are needed to estimate the importance weight of each criterion (Table 1) and measure the relationships between the critical success factors and the fuzzy rates of alternatives assigned by decision makers (Table 2).

Phase 1: Applying DEMATEL for constructing the interdependence relationship network

Step 1: find the initial direct-relation matrix. To measure the relationships between the critical success factors which are demonstrated by $C = \{Ci \mid i = 1,2...n\}$, the group of the chosen experts is asked to express the degree which the criterion i affects the criterion j in terms of linguistic terms. Score given by each expert, leading to a matrix $\tilde{Z}^{(k)} = \left[\tilde{z}_{ij}^{(k)}\right] n \times n$. Thus, $\tilde{Z}^{(1)}, \tilde{Z}^{(2)}, ..., \tilde{Z}^{(p)}$, are matrix related to each expert obtained with triangular fuzzy numbers. Denote $\tilde{Z}^{(k)}$ as:

$$\tilde{Z}^{(k)} = \begin{bmatrix} 0 & \tilde{z}_{12}^{(k)} & \cdots & \tilde{z}_{1n}^{(k)} \\ \tilde{z}_{21}^{(k)} & 0 & \cdots & \tilde{z}_{2n}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{z}_{n1}^{(k)} & \tilde{z}_{n2}^{(k)} & \cdots & 0 \end{bmatrix}; k = 1,2,...,p, \quad (3)$$

$$\tilde{z}_{ij}^{(k)} = (l_{ij}^{(k)}, m_{ij}^{(k)}, u_{ij}^{(k)}),$$

Step 4: produce the normalized direct-relation fuzzy matrix. First consider $\tilde{a}_i^{(k)}$ and $r^{(k)}$ as triangular fuzzy numbers as follows:

$$\tilde{a}_i^{(k)} = \sum \tilde{z}_{ij}^{(k)} = \left(\sum_{j=1}^{n} l_{ij}^{(k)}, \sum_{j=1}^{n} m_{ij}^{(k)}, \sum_{j=1}^{n} u_{ij}^{(k)} \right) \quad (4)$$

$$r^{(k)} = \max\left(\sum_{j=1}^{n} u_{ij}^{(k)} \right), \quad 1 \le i \le n \quad (5)$$

Then the linear scale transformation is used to transform the criteria scales into comparable scales. The normalized direct relation fuzzy matrix of experts denotes as $\tilde{X}^{(k)}$ is given by:

$$\tilde{X}^{(k)} = \begin{bmatrix} \tilde{x}_{12}^{(k)} & \tilde{x}_{12}^{(k)} & \cdots & \tilde{x}_{1n}^{(k)} \\ \tilde{x}_{21}^{(k)} & \tilde{x}_{22}^{(k)} & \cdots & \tilde{x}_{2n}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{n1}^{(k)} & \tilde{x}_{n2}^{(k)} & \cdots & \tilde{x}_{nn}^{(k)} \end{bmatrix}, \ k = 1,2,...,p \quad (6)$$

Where $\tilde{x}_{ij}^{(k)} = \frac{\tilde{z}_{ij}^{j}}{r^{k}} = \left(\frac{l_{ij}^{(k)}}{r^{k}}, \frac{m_{ij}^{(k)}}{r^{k}}, \frac{u_{ij}^{(k)}}{r^{k}} \right)$, (7)

As same as the crisp DEMATEL method, we assume at least one i such that $\sum_{j=1}^{n} u_{ij}^{(k)} < r^{(k)}$. This assumption is adaptable with the real and practical cases. To calculate the average matrix of $\tilde{X}$, we use Eqs. 8 and 9 as follows:

$$\tilde{X} = \frac{\left(\tilde{x}^{(1)} + \tilde{x}^{(2)} + ... + \tilde{x}^{(p)} \right)}{p}, \quad (8)$$

$$\tilde{X} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{nn} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{n1} & \tilde{x}_{n2} & \cdots & \tilde{x}_{nn} \end{bmatrix}; \text{ Where } \tilde{x}_{ij} = \frac{\sum_{k=1}^{p} \tilde{x}_{ij}^{(k)}}{p} \quad (9)$$

Step 5: Establish and analyze the structured model. So to compute the total-relation fuzzy matrix $\tilde{T}$, we should have to ensure about the convergence of $\lim_{w \to \infty} \tilde{X}^{w} = 0$. According to the crisp case we define the total-relation fuzzy matrix as:

$$\tilde{T} = \lim_{w \to \infty} (\tilde{X} + \tilde{X}^{2} + ... + \tilde{X}^{w}), \quad (10)$$

$$\tilde{T} = \begin{bmatrix} \tilde{t}_{11} & \tilde{t}_{12} & \cdots & \tilde{t}_{1n} \\ \tilde{t}_{21} & \tilde{t}_{22} & \cdots & \tilde{t}_{nn} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{t}_{n1} & \tilde{t}_{n2} & \cdots & \tilde{t}_{nn} \end{bmatrix}, \quad (11)$$

Where $\tilde{t}_{ij} = (l''_{ij}, m''_{ij}, u''_{ij})$,

$$Matrix\left[l''_{ij} \right] = X_{l} \times (l - X_{l})^{-1},\ Matrix\left[m''_{ij} \right] = X_{m} \times (l - X_{m})^{-1},\ Matrix\left[u''_{ij} \right] = X_{u} \times (l - X_{u})^{-1}, \quad (12)$$

The amount $Matrix\left[l''_{ij} \right], Matrix\left[m''_{ij} \right], Matrix\left[u''_{ij} \right]$ and finally matrix $\tilde{T}$ are mentioned above.

Step 6: After computing the matrix $\tilde{T}$, now it is easy to calculate the amounts of $(\tilde{D}_i + \tilde{R}_i)$ and $(\tilde{D}_i - \tilde{R}_i)$, the sum of each column and row of matrix T is respectively marked as vectors $\tilde{D}_i$ and $\tilde{R}_i$ in matrix $\tilde{T}$, a level of influence to others and a level of relationship with others are defined.

$$\tilde{R} = (\sum_{j=1}^{n} \tilde{t}_{ij})_{n*1} \quad (13)$$

$$\tilde{D} = (\sum_{I=1}^{n} \tilde{t}_{ij})_{1*n} \quad (14)$$

To acquire the causal diagram, after diffuzificate the amount of $(\tilde{D}_i + \tilde{R}_i)$ and $(\tilde{D}_i - \tilde{R}_i)$ and convert to $(\tilde{D}_i + \tilde{R}_i)^{def}$ and $(\tilde{D}_i - \tilde{R}_i)^{def}$, respectively. Each triangular fuzzy number is then defuzzied into a crisp number by the COG method by Eq. (2).

Phase 2: ANP for calculating the sub-criteria weights

Steps in the ANP process can be divided as following:

Step 1: Perform the pairwise comparisons. Before performing the pairwise comparisons, all criteria and clusters compared are connected to each other. In the application for performing the pairwise comparisons among attributes, triangular fuzzy numbers have been used by experts to state their preferences according Table 1. Pairwise comparison matrices are structured by using triangular fuzzy numbers (l, m, u) [23]. The a_{mn} represents the of comparison m (row) with component *n* (column). The pair-wise comparison matrix $\tilde{A}$ is assumed as reciprocal:

$$\tilde{A} = \begin{bmatrix} (1,1,1) & (a_{12}^{l}, a_{12}^{m}, a_{12}^{u}) & \cdots & (a_{1n}^{l}, a_{1n}^{m}, a_{1n}^{u}) \\ (\frac{1}{a_{12}^{u}}, \frac{1}{a_{12}^{m}}, \frac{1}{a_{12}^{l}}) & (1,1,1) & \cdots & (a_{2n}^{l}, a_{2n}^{m}, a_{2n}^{u}) \\ \vdots & \vdots & \vdots & \vdots \\ (\frac{1}{a_{1n}^{u}}, \frac{1}{a_{1n}^{m}}, \frac{1}{a_{1n}^{l}}) & (\frac{1}{a_{2n}^{u}}, \frac{1}{a_{2n}^{m}}, \frac{1}{a_{2n}^{l}}) & \cdots & (1,1,1) \end{bmatrix}, \qquad (15)$$

Step 2: construct aggregated fuzzy pairwise comparison matrix. By considering K experts (DMs), every pairwise comparison between two elements has K positive triangular fuzzy numbers. The triangular fuzzy number $\tilde{a}_{ij} = \left(a_{ij}^{l}, a_{ij}^{m}, a_{ij}^{u}\right)$ as the aggregated group of the individual judgment by all K DMs is calculated using the geometric average approach shown as:

$$a_{ij}^{l} = \min\left\{a_{ij1}^{l}, a_{ij2}^{l}, ..., a_{ijk}^{l}\right\},$$

$$a_{ij}^{m} = \left(\prod_{k=1}^{k} a_{ijk}^{m}\right)^{1/k},$$

$$a_{ij}^{u} = \max\left\{a_{ij1}^{u}, a_{ij2}^{u}, ..., a_{ijk}^{u}\right\},, \qquad (16)$$

Step 3: compute the local weight vector and determine defuzzied weights. After forming the fuzzy pairwise comparison matrix, the logarithmic least squares method can used for calculating triangular fuzzy weight $\tilde{w}_j$ as follows:

$$W = (W_k^{\,l}, W_k^{\,m}, W_k^{\,u}) \qquad k = 1, 2, ..., n, \qquad (17)$$

Where

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$$W_k^s = \frac{(\prod_{j=1}^{n} a_{kj}^s)^{1/n}}{\sum_{i=1}^{n}(\prod_{j=1}^{n} a_{ij}^m)^{1/n}}, \quad s \in \{i, m, u\}. \quad (18)$$

Each triangular fuzzy number is then defuzzied into a crisp number by the COG method by Eq. (2).
Step 4: Construct of the un-weighted super-matrix. To determine global priorities in a system by considering interdependent influences, the local priority vectors are entered in the appropriate columns of a matrix, known as an un-weighted super-matrix [23]. The super-matrix representation of a network with three levels is given as follows.

$$W = \begin{matrix} Goal(G) \\ Criteria(C) \\ Subcriteria(S) \end{matrix} \begin{bmatrix} 0 & 0 & 0 \\ W_{21} & W_{22} & 0 \\ 0 & W_{32} & I \end{bmatrix}, \quad (19)$$

W displays the structure and super-matrix in a network. The super-matrix with three levels of clusters is also shown in where W_{21} is a vector that represents the impact of the goal on the criteria; and W_{32} is a matrix that represents the impact of the criteria on each of sub-criteria; in the super-matrix middle, W_{22} would indicate the interdependency and *I* is the identity matrix [24].
Step 5: Transform un-weighted super-matrix into weighted super-matrix. An eigenvector is obtained from the pair-wise comparison matrix of the row clusters with respect to the column cluster, which in turn yields an eigenvector for each column cluster. The first entry of the respective eigenvector for each column cluster is multiplied by all the elements in the first cluster of that column, the second by all the elements in the second cluster of that column and so on. In this way, the cluster in each column of the super-matrix is weighted, and the result, known as the weighted super-matrix, is stochastic [25].
Step 6: Compute the limit super-matrix. The weighted super-matrix is raised to the power of 2 h + 1 to reach the limit super-matrix.
In end, the weights of sub-criteria are obtained for the fuzzy multi-criteria analysis.

Phase 3: TOPSIS method for calculating the overall weight

TOPSIS assumes that there are m alternatives and n attributes (criteria). Given that there are scores of each alternative with respect to each attribute, we can adopt the following procedure for finding the weights.
Let $\tilde{x}_{ijk} = (a_{ijk}, b_{ijk}, c_{ijk}); \{i = 1,2,3,...,n, j = 1,2,3,...,m\}$

That it is the rating of the kth decision maker for alternative *j* with respect to criterion *i*. Hence, the aggregated fuzzy ratings $\tilde{x}_{ijk}$ of alternatives with respect to each criterion can be given as following:

$$\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}) \text{ Where } a_{ij} = \min_k\{a_{ijk}\},\ b_{ij} = \frac{1}{k}\sum_{k=1}^{K} b_{ijk},\ c_{ij} = \max_k\{c_{ijk}\}. \quad (20)$$

Output of the ratings will be a matrix $X = (\tilde{x}_{ij})$ can be briefly illustrated in following format:

$$X = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{12} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{32} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{j1} & \tilde{x}_{j2} & \cdots & \tilde{x}_{jn} \end{bmatrix} \quad (21)$$

To select the supplier rating by TOPSIS, The following steps can be summarized as follows [24, 26]
Step 1: Construct normalized decision matrix.

Choose the fuzzy ratings $(\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}), i = 1, 2, ..., n, j = 1, 2, ..., m)$ for alternatives with respect to criteria as formerly stated. Here in a decision process for avoiding complication of mathematical operations, the linear scale transformation is used to convert the various criteria scales into comparable scales. The set of criteria can be divided into benefit criteria and cost criteria. Let $\tilde{x}_i^* = (a_i^*, b_i^*, c_i^*)$ and $\tilde{x}_i^- = (a_i^-, b_i^-, c_i^-)$; Get K and K' are the sets of benefit criteria and cost criteria, respectively, we have:

$$\tilde{r}_{ij} = \tilde{x}_{ij} / \tilde{x}_i^* = (\frac{a_{ij}}{a_i^*}, \frac{b_{ij}}{b_i^*}, \frac{c_{ij}}{c_i^*}), \text{ Where } a_i^* = \max_j a_{ij}, b_i^* = \max_j b_{ij}, a_i^* = \max_j b_{ij}, \ i \in K, \quad (22)$$

$$\tilde{r}_{ij} = \tilde{x}_i^- / \tilde{x}_{ij} = (\frac{a_i^-}{a_{ij}}, \frac{b_i^-}{b_{ij}}, \frac{c_i^-}{c_{ij}}), \text{ Where } a_i^- = \min_j a_{ij}, b_i^- = \min_j b_{ij}, a_i^- = \min_j b_{ij}, \ i \in K', \quad (23)$$

Step 2: Calculate the weighted normalized decision matrix.
Assume that we have a set of weights for each criterion w_i for ($\text{i} = 1, 2, 3, \ldots, \text{n}$). Where w_i is the weight of the *i*th criterion. Multiply each column of the normalized decision matrix by its related weight. The new matrix is called $\tilde{V}_{ij}$. Therefore the value of $\tilde{v}_{ij}$ is obtained as following:

$$\tilde{V}_{ij} = [\tilde{v}_{ij}]_{nm} \quad \{i = 1, 2, ..., n, j = 1, 2, ..., m\} \text{ Where } \tilde{v}_{ij} = \tilde{r}_{ij}.w_i . \quad (24)$$

Step 3: Determine the positive and negative ideal solutions
As mentioned earlier, positive ideal solutions (A^*) are near to the best alternative and negative ideal solutions (A^-) are farthest from the alternatives. Ideal solution is given by:

$$A^* = \{\tilde{v}_1^*, \tilde{v}_2^*, ..., \tilde{v}_i^*\}, \text{ Where } \tilde{v}_i = \{(\max_j v_{ij} \ \ \text{I} i \in K), (\min_j v_{ij} \ \ \text{I} i \in K')\} \quad (25)$$

Similarly negative ideal solution is given by,

$$A^- = \{\tilde{v}_1^-, \tilde{v}_2^- ..., \tilde{v}_i^-\}, \text{ Where } \tilde{v}_i = \{(\min_j v_{ij} \ \ \text{I} i \in K), (\max_j v_{ij} \ \ \text{I} i \in K')\} \quad (26)$$

Step 4: Calculate the distance of each alternative from A^* and A^- .

Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. Then the distance between them can be determined by using the vertex method [27].

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2)]} \quad (27)$$

By the formula, the two distances for each alternative are respectively calculated as:

$$D_j^* = \sum_{i=1}^{n} d(\tilde{v}_{ij}, v_i^*) \ \{j = 1, 2, 3, ..., m\}, \quad (28)$$

$$D_j^- = \sum_{i=1}^{n} d(\tilde{v}_{ij}, v_i^-) \; \{j = 1,2,3,...,m\}, \qquad (29)$$

Step 5: Calculate the relative closeness to the ideal solution.

A closeness coefficient is defined to determine the ranking order of all possible alternatives by using D_j^* and D_j^- of each alternative. The closeness coefficient (CC_j) of each alternative can be defined as:

$$CC_j = \frac{D_j^-}{D_j^- + D_j^*}, \; \{j = 1,2,3,...,m\}, \qquad (30)$$

Thus the best alternative can be selected with CC_i closest to 1. In other words, the higher the closeness means the better the rank. Therefore, the best alternative can be selected from among a set of possible alternatives.

5. An Example of a Supplier Selection Problem

For implementing the supplier selection model, a committee of four decision makers (DM_1, DM_2, DM_3 and DM_4) is established including managers from different functional divisions bringing particular concerns and knowledge into the evaluation. Committee starts its work with anticipation and definition of the evaluation criteria. These major criteria usually interdependent on each other in the decision making process are provided in Table 3. Also, the committee chooses four suppliers (V_1, V_2, V_3 and V_4) as initial candidate for further evaluation based the preliminary screening.

Table 3. Criteria and sub-criteria for the supplier selection

(Q)	**Quality**
(PQ)	Product quality
(SS)	Service satisfaction
(P & DC)	**Price & deliver condition**
(PP)	Product price
(R)	Responsiveness
(DO)	Delivery operation
(DT)	Delivery time
(SCS)	**Supply chain support**
(POR)	Purchase order reactiveness
(CSF)	Capacity support & flexibility
(T)	**Technology**
(TS)	Technical support
(DI)	Design involvement

Because of the interdependence among the criteria, in first phase the committee should determine the network relationships among criteria in respect to their influences on each other. To measure the relationships between the critical success factors, the decision makers was asked to make sets of pair wise comparisons in terms of linguistic terms Table 2. Hence 4 fuzzy matrices $\tilde{Z}^{(1)}, \tilde{Z}^{(2)}, \tilde{Z}^{(3)}, \tilde{Z}^{(4)}$ are corresponding to an expert and with triangular fuzzy numbers as its elements are obtained. Then, a triangular fuzzy number ($\tilde{a}_i^{(k)}$) according to equations 4 and 5 is considered to calculate each direct–relation fuzzy matrix $\tilde{X}^{(k)}$ for each matrix $\tilde{Z}^{(k)}$. For example for matrix $\tilde{Z}^{(1)}$, the normalized direct relation fuzzy matrix $\tilde{X}^{(1)}$ can be calculated by Eqs. 4 and 5 as follows:

The amount of $\tilde{X}$ (The total normalized direct-relation fuzzy matrix) is calculated by the Eqs. 6 and 7. The procedure of calculation matrix $\tilde{T}$ (The total relation matrix) according to the Equations 10 and 11 is calculated. After computing the matrix $\tilde{T}$, the amounts of $(\tilde{D}_i + \tilde{R}_i)$ and $(\tilde{D}_i - \tilde{R}_i)$ are calculated.

We use the Eq. 2 for diffuzification of the amount of $\tilde{D}_i + \tilde{R}_i$ and $\tilde{D}_i - \tilde{R}_i$ and convert to $(\tilde{D}_i + \tilde{R}_i)^{def}$ and $(\tilde{D}_i - \tilde{R}_i)^{def}$ respectively.

Table 4. The Initial direct matrix

Criteria	Q	P & DC	SCS	T
Q	0	1.88	2.22	2
P & DC	2.22	0	5.16	2.33
SCS	1.167	1.44	0	1.267
T	0.667	0.867	0.667	0

Table 5. The normalized direct-relation matrix

Criteria	Q	P & DC	SCS	T
Q	0.000	0.190	0.225	0.203
P & DC	0.225	0.000	0.523	0.236
SCS	0.121	0.148	0.000	0.331
T	0.069	0.070	0.069	0.000

Table 6. The total-relation matrix

Criteria	Q	P & DC	SCS	T	D	D+R	D-R
Q	0.673	0.904	1.05	0.883	3.51	7.444	-0.424
P & DC	1.213	1.135	1.606	1.28	5.234	9.58	-0.888
SCS	0.23	0.35	0.702	0.3	2.082	4.915	0.249
T	0.018	0.057	0.075	0.115	1.465	3.643	0.713
R	3.934	4.346	2.833	2.178			

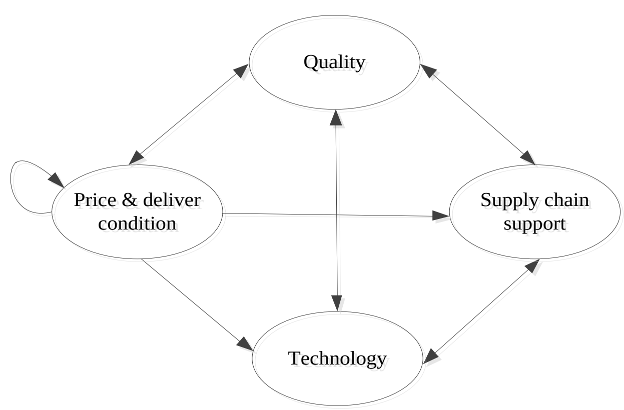

Fig. 1 Network relationship map of impacts for the supplier selection problem

In the second phase, four decision makers used the linguistic variables shown in Table 1 to assess the importance of four criteria by making pair-wise comparison in respect to the goal, and then fuzzy variable are converted into the triangular fuzzy numbers according Table 1. Forming the pairwise comparison matrix, the weight vector (i.e., calculate W_{21}) can be calculated by the logarithmic least squares method. Moreover, the fuzzy interdependences and feedback among the criteria are subsequently specified based on the linguistic evaluation. By using the logarithmic least squares method again, triangular fuzzy importance weights are derived and these weights are arranged into the fuzzy interdependence matrix (i.e., calculate W_{22}). The data for the fuzzy feedbacks among the criteria is composed of the four pair-wise comparison matrices for each criterion. Table 7 summarizes the pair-wise comparison of the four criteria with respect to the overall goal and the four criteria (W_{21}, W_{22}).

Table 7. Aggregated fuzzy pair wise comparison matrix of the four criteria with respect to the overall goal and the four criteria (inner dependence)

	Q	P & DC	SCS	T	Crisp weights
With respect to sustainable supplier selection (Goal)					
Q	(1, 1, 1)	(0.17, 0.29, 0.5)	(4, 6.3, 10)	(2, 4.21, 8)	0.29
P & DC	(2, 3.4, 6)	(1, 1, 1)	(6, 7.94, 10)	(2, 5.66, 8)	0.57
SCS	(0.1, 0.16, 0.25)	(0.1, 0.13, 0.17)	(1, 1, 1)	(0.25, 0.33, 0.5)	0.05
T	(0.12, 0.24, 0.5)	(0.12, 0.18, 0.5)	(2, 3, 4)	(1, 1, 1)	0.11
With respect to quality (Q)					
P & DC		(1, 1, 1)	(4, 6.85, 10)	(2, 5.66, 8)	0.71
SCS		(0.1, 0.15, 0.25)	(1, 1, 1)	(2, 3.41, 6)	0.18
T		(0.12, 0.18, 0.5)	(0.17, 0.29, 0.5)	(1, 1, 1)	0.09
With respect to price & deliver condition (P & DC)					
Q	(1, 1, 1)	(0.12, 0.27, 0.5)	(4, 6.3, 10)	(6, 7.94, 10)	0.30
P & DC	(2, 3.7, 8)	(1, 1, 1)	(4, 6.43, 8)	(6, 8.45, 10)	0.59
SCS	(0.1, 0.16, 0.25)	(0.12, 0.16, 0.25)	(1, 1, 1)	(2, 3.87, 6)	0.09
T	(0.1, 0.13, 0.17)	(0.1, 0.12, 0.17)	(0.17, 0.26, 0.5)	(1, 1, 1)	0.04
With respect to supply chain support (SCS)					
Q	(1, 1, 1)			(4, 5.44, 8)	0.87
T	(0.12, 0.18, 0.25)			(1, 1, 1)	0.15
With respect to technology (T)					
Q	(1, 1, 1)		(4, 6.3, 10)		0.88
SCS	(0.1, 0.16, 0.25)		(1, 1, 1)		0.14

Then, local weights of the sub-criteria (W_{32}) were determined by using the pairwise comparison matrices listed in Table 8-11.

Table 8. Aggregated local weights of sub criteria for the Q

	PQ	SS	Crisp weights
PQ	(1, 1, 1)	(1, 2.6, 6)	0.76
SS	(0.17, 0.38, 1)	(1, 1, 1)	0.30

Table 9. Aggregated local weights of sub criteria for the P & DC

	PP	R	DO	DT	Crisp weights
PP	(1, 1, 1)	(1, 2.6, 6)	(2, 4.21, 10)	(2, 4.79, 10)	0.59
R	(0.17, 0.38,1)	(1, 1, 1)	(1, 2.6, 6)	(2, 3.7, 8)	0.31
DO	(0.1, 0.24, 0.5)	(0.17, 0.38, 1)	(1, 1, 1)	(1, 2.6, 6)	0.15
DT	(0.1, 0.21, 0.5)	(0.12, 0.27, 0.5)	(0.17, 0.38, 1)	(1, 1, 1)	0.06

Table 10. Aggregated local weights of sub criteria for the SCS

	POR	CSF	Crisp weights
POR	(1, 1, 1)	(1, 2.14, 8)	0.82
CSF	(0.12, 0.47, 1)	(1, 1, 1)	0.32

Table 11. Aggregated local weights of sub criteria for the T

	TS	DI	Crisp weights
TS	(1, 1, 1)	(2, 3.87, 6)	0.78
DI	(0.17, 0.26, 0.5)	(1, 1, 1)	0.22

Also third phase of the study called the fuzzy TOPSIS, the decision makers use the linguistic rating variables shown in Table 2 to evaluate the ratings of candidate suppliers (A_1, A_2, A_3.,A_4) with respect to each criterion. Then, by getting the aggregated fuzzy weights of suppliers, fuzzy decision matrix is constructed as indicating the performance ratings of the alternatives (Table 12). After forming the decision matrix, a normalized weighted decision matrix is calculated by using the sub-criteria weights derived from fuzzy ANP. This matrix is shown in Table 13.

Table 12. Aggregated decision matrix for performance evaluation of 4 suppliers

	PQ	SS	PP	R	DO
A1	(0.10,0.44,0.70)	(0.30,0.54,0.90)	(0.50,0.77,1)	(0.70,0.92,1)	(0.50,0.85,1)
A2	(0.70,0.95,1)	(0.30,0.69,1)	(0.30,0.75,1)	(0.30,0.58,1)	(0.50,0.89,1)
A3	(0.30,0.67,1)	(0.50,0.79,1)	(0.50,0.75,1)	(0.30,0.63,1)	(0.30,0.59,1)
A4	(0.30,0.63,1)	(0.10,0.52,0.90)	(0.30,0.77,1)	(0.10,0.52,0.90)	(0.30,0.54,.090)
	DT	POR	CSF	TS	DI
A1	(0.70,0.92,1)	(0.50,0.87,1)	(0.30,0.75,1)	(0.30,0.67,1)	(0.50,0.87,1)
A2	(0.10,0.57,1)	(0.50,0.84,1)	(0.50,0.79,1)	(0.30,0.58,1)	(0.30,0.80,1)
A3	(0.30,0.73,1)	(0.30,0.58,1)	(0.30,0.75,1)	(0.30,0.69,1)	(0.30,0.65,1)
A4	(0.10,0.48,090)	(0.30,0.67,1)	(0.30,0.59,090)	(0.50,0.79,1)	(0.10,0.52,090)

Table 13. The weighted normalized decision matrix

	Weight	A_1	A_2	A_3	A_4
(PQ)	0.175	(0.052,0.043,0.082)	(0.206,0.206,0.206)	(0.103,0.167,0.206)	(0.103,0.085,0.124,)
(SS)	0.061	(0.042,0.058,0.083)	(0.083,0.083,0.083)	(0.083,0.083,0.083)	(0.042,0.041,0.062)
(PP)	0.281	(0.143,0.176,0.215)	(0.286,0.286,0.286)	(0.072,0.118,0.172)	(0.143,0.166,0.172)
(R)	0.128	(0.076,0.054,0.091)	(0.151,0.151,0.151)	(0.076,0.047,0.060)	(0.076,0.071,0.091)
(DO)	0.065	(0.019,0.029,0.060)	(0.075,0.075,0.075)	(0.019,0.027,0.045)	(0.038,0.057,0.075)
(DT)	0.039	(0.005,0.004,0.012)	(0.031,0.012,0.016)	(0.008,0.005,0.012)	(0.031,0.031,0.031)
(POR)	0.137	(0.033,0.046,0.040)	(0.066,0.066,0.066)	(0.033,0.044,0.053)	(0017,0.027,0.040)

Moreover the fuzzy positive ideal solution (A^*) and the fuzzy negative ideal solution (A^-) are calculated using Eqs. (25) and (26). The distance of each alternative from A^* and A^- is computed by using Eqs. (28) and (29). In the end, closeness to the ideal solution are calculated and ranked in preference orders using Eq. (30). An alternative with maximum CC_j is chosen or alternatives according to CC_j are ranked in descending order. According to the Fuzzy TOPSIS, the best alternative for the supplier selection problem is determined as A_2. The alternatives are ranked as A_2, A_1, A_4 and A_5, respectively, shown in Table 14.

Table 14. Computations of overall scores of the 4 suppliers and their priority ranking

	d_j^+	d_j^-	Cl_j	Priority ranking
A_1	0.492	0.479	0.49	2
A_2	0.056	0.558	0.91	1
A_3	0.427	0.362	0.46	4
A_4	0.464	0.321	0.41	3

6. Conclusion

In this paper, we for solving supplier selection problem have proposed a new integrated hybrid MCDM methodology combined DEMATEL, ANP and TOPSIS. Linguistic terms (such as high, medium and low) is applied to all techniques in order to make the evaluation process more precise and more flexible for assessing suppliers under each criterion. In other words, using linguistic preferences can be very useful for uncertain situations. Due to the fact that criteria of our issue are interdependent on each other in practice, we introduced the fuzzy analytic network process (ANP) for obtaining a set of suitable weights of the sub-criteria. Also DEMATEL method is applied as a supportive tool for ANP to construct the structure of relationship map for clarifying the interrelations among criteria. The fuzzy TOPSIS is employed to rank competing suppliers in terms of their overall performance with multiple sub-criteria. Important feature of the technique is to allows explicit trade-offs and interactions among attributes.
Various different kinds of MCDM methods can be employed in future studies regarding supplier selection issue. As a future work to this paper could be the comparison of the proposed approach to other MCDM methods, like VIKOR, ELECTRE and even AHP.

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